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Title:	Nonreciprocal Metasurfaces
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Nonreciprocal metasurfaces

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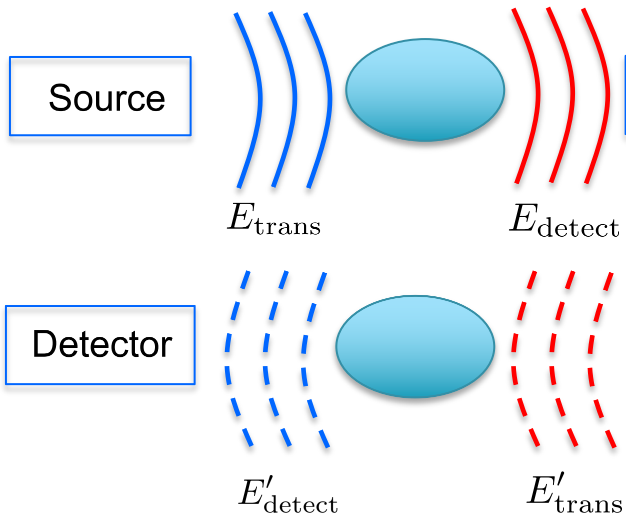
Outline

- ❑ Introduction to electromagnetic reciprocity
- ❑ Nonreciprocal EM systems
- ❑ Time-Floquet metasurfaces
- ❑ Spatio-temporal modulated metasurfaces for nonreciprocal wavefront control
- ❑ Summary and future work

Electromagnetic reciprocity

- Reciprocity: "going the same way backward as forward". A reciprocal system exhibits the same received-transmitted field ratios when it source(s) and detector(s) are exchanged



$$\eta = \frac{E_{\mathrm{detect}}}{E_{\mathrm{trans}}}$$

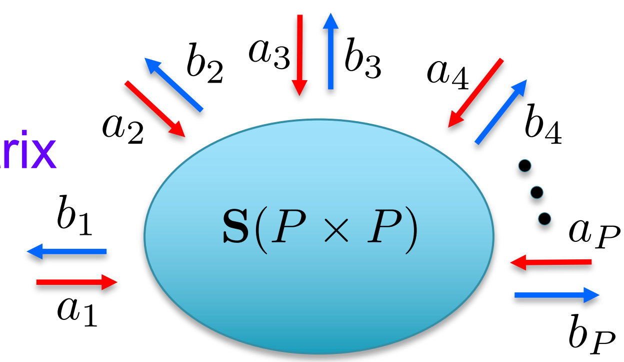
$$\eta = \eta' \rightarrow \mathrm{Reciprocity}$$

$$\eta' = \frac{E'_{\mathrm{detect}}}{E'_{\mathrm{trans}}}$$

- A reciprocal system has a symmetric scattering matrix

$$\mathbf{S} = \mathbf{S}^{\mathrm{T}} \text{ or } S_{ij} = S_{ji} \ \forall (i,j)$$

  - For linear systems $S_{ij} = b_i/a_j|_{a_k=0, k\neq j}$
  - For nonlinear systems $S_{ij} = S_{ij}(a_1, a_2, \ldots, a_P)$

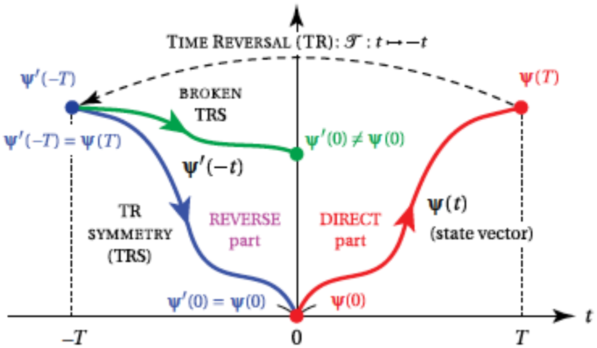


$$\mathbf{b} = \mathbf{S}\ \mathbf{a}$$

- Linear, time-independent, symmetric $\boldsymbol{\epsilon}$ and $\boldsymbol{\mu}$ → reciprocity

Reciprocity and time-reversal symmetry (TRS)

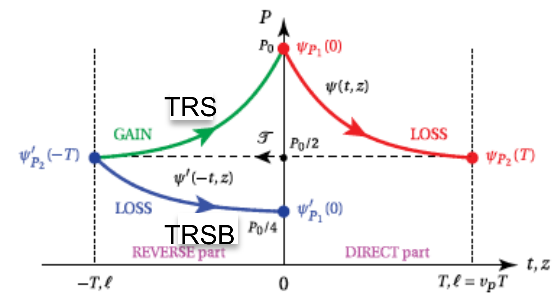
❑ TRS $\Rightarrow$ Reciprocity



but the converse is not true

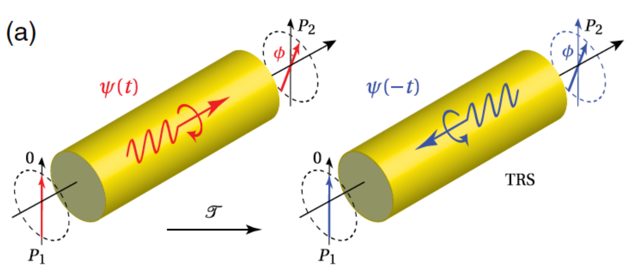
❑ Reciprocity $\not\Rightarrow$ TRS

(e.g. lossy systems)



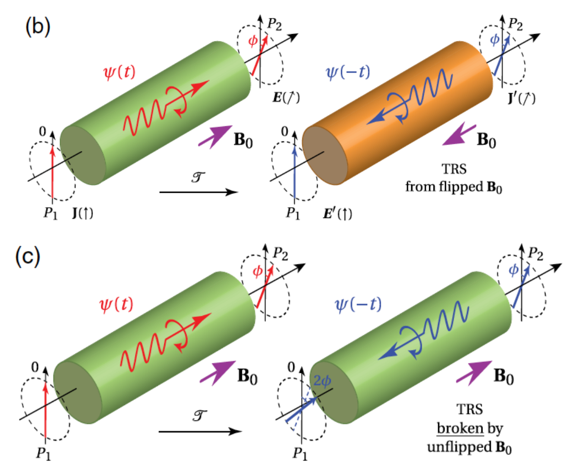
$$(P_0/4)/(P_0/2) = 0.5 = (P_0/2)/P_0$$

❑ Chiral systems are reciprocal



$$\epsilon_{yx} = \epsilon_{xy}$$

❑ Faraday systems are not reciprocal



System altered on full TR, so not useful for properly deciding on reciprocity

Preserve spin states by not flipping $\mathbf{B}_0$

$$\mu_{yx}(\mathbf{B}_0) = -\mu_{xy}(\mathbf{B}_0) \qquad \epsilon_{yx}(\mathbf{B}_0) = -\epsilon_{xy}(\mathbf{B}_0)$$

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Electromagnetic nonreciprocity

- **Nonreciprocity:** absence of reciprocity! $\mathbf{S}(\mathbf{F}_0) \neq \mathbf{S}^T(\mathbf{F}_0)$ or $\exists(i,j) \mid S_{ij} \neq S_{ji}$

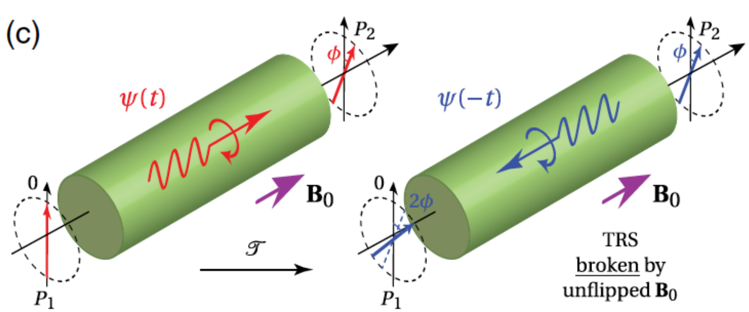
- **Lorentz theorem** suggests a few directions in the quest for nonreciprocity

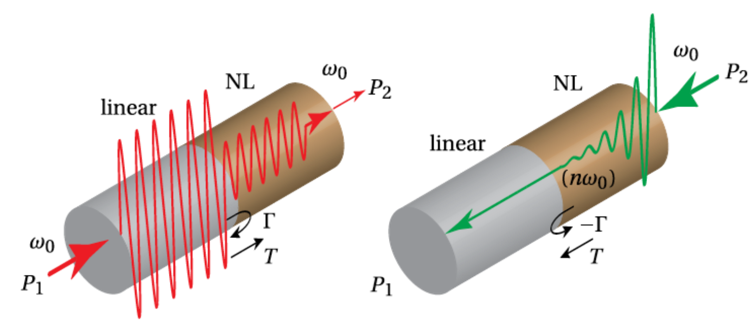
Magneto-optical materials

- External bias: magnetic field
- Linear (strong nonreciprocity)
- Time-invariant (frequency conservation)
- Ferrites, 2D electron gases, etc.
- Require bulky magnets

Nonlinear materials

- Self-biasing (electric field) + spatial asymmetry
- Harmonic generation
- Inapplicability of superposition
- Power dependent (weak nonreciprocity)

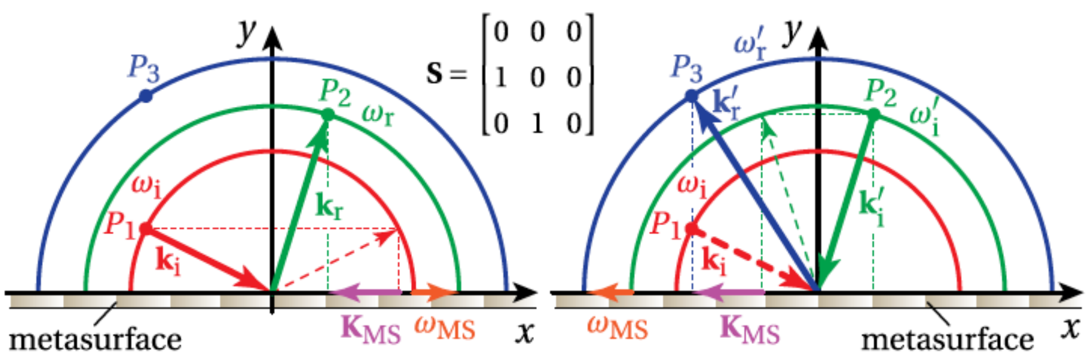




Spatio-temporal modulation for nonreciprocity

➢ Space-time modulation

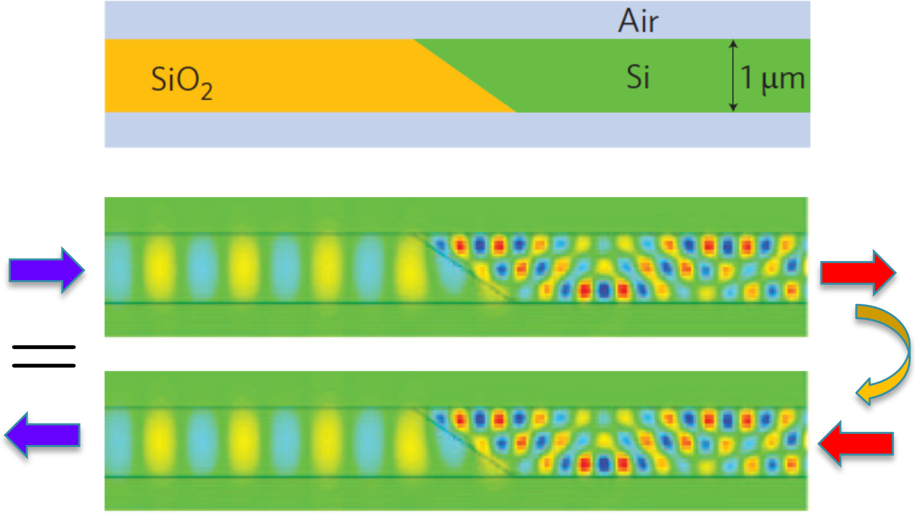
- External bias: velocity (~ spatial inversion symmetry breaking)
- Linear (strong nonreciprocity)
- Harmonic & anharmonic generation
- Optomechanics, electro/acousto-optics, STMMs
- Pulse or periodic, abrupt or smooth medium/wave modulation
- Challenging implementation!

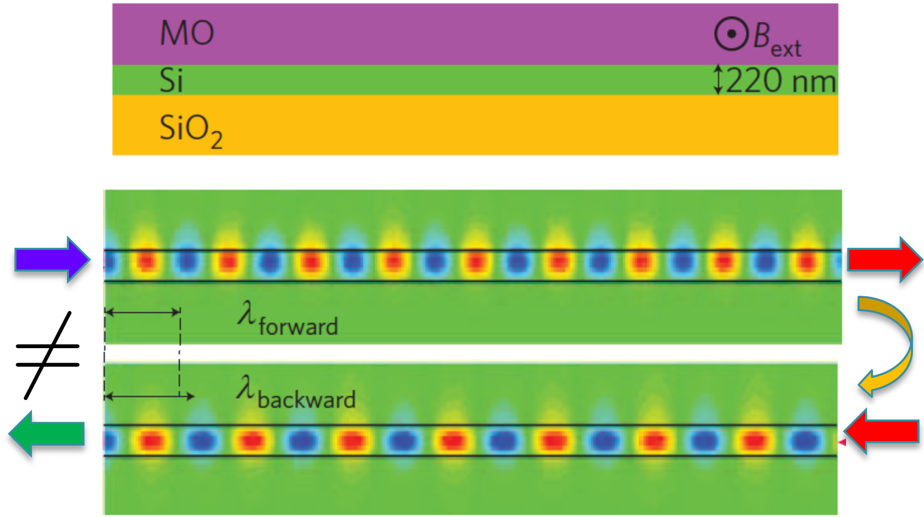


$$n(x) = n_0 + \delta n \cos(K_{\text{MS}} x + \omega_{\text{MS}} t)$$

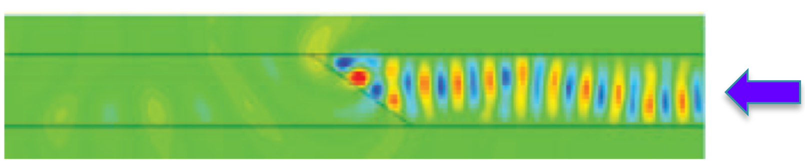
Nonreciprocity is different from asymmetric propagation

- There is a large confusion in the literature between nonreciprocity and asymmetric propagation





- System is reciprocal (not an isolator)
- But, there is asymmetric propagation



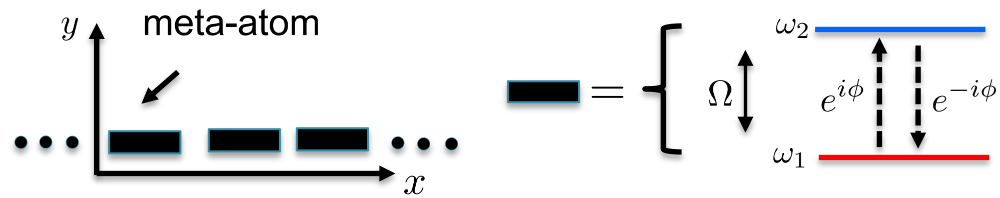
- System is nonreciprocal (can be used as an isolator)
- Asymmetric scattering matrix

$$S_{12}(B) \neq S_{21}(B)$$

Time-modulated metasurfaces

❑ Time-modulated meta-atom

$$\boxed{\epsilon(x,y,t) = \epsilon_c(x,y) + \delta\epsilon(x,y,t)}$$



e.g. $\delta\epsilon = \delta(x,y)\cos(\Omega t + \phi)$

Assume EM field is z-polarized. We need to solve

$$\nabla^2 E(x,y,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left[\epsilon(x,y,t)E(x,y,t)\right]$$

❑ Coupled mode theory

$E(x,y,t) = a_1(t)E_1(x,y)a_2(t)E_2(x,y)$ where $\nabla^2 E_{1,2}(x,y) = -\epsilon_c(x,y)\dfrac{\omega_{1,2}^2}{c^2}E_{1,2}(x,y)$

Assuming small and slow perturbation, i.e. $\delta\epsilon/\epsilon_c,\ \dot{\delta\epsilon}/\omega_{1,2}\epsilon_c,\ \ddot{\delta\epsilon}/\omega_{1,2}^2\epsilon_c \ll 1$

$$\boxed{\begin{aligned}\dot{a}_1(t) &= (i\omega_1 - \gamma_1)a_1(t) + iC_{12}(t)a_2(t)\\ \dot{a}_2(t) &= (i\omega_2 - \gamma_2)a_2(t) + iC_{21}(t)a_1(t)\end{aligned}}$$

$$C_{12}(t) = \mathcal{C}\int_{\mathrm{MA}} dxdy\ E_1^*(x,y)E_2(x,y)\delta\epsilon(x,y,t)$$

$$C_{21}(t) = \mathcal{C}\int_{\mathrm{MA}} dxdy\ E_1(x,y)E_2^*(x,y)\delta\epsilon(x,y,t)$$

Note that $\delta\epsilon$ must break spatial symmetry; otherwise $C_{12} = C_{21} = 0$

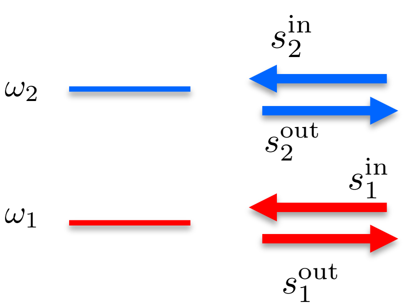
Harmonic modulation of the complex dielectric function

❑ Harmonic time modulated perturbation

$$\boxed{\delta\epsilon(x,y,t) = \delta_R(x,y)\cos(\Omega t + \phi_R) + i\delta_I(x,y)\sin(\Omega t + \phi_I)}$$

$$\Rightarrow C_{12}(t) = C_{12}^{+}e^{i\Omega t} + C_{12}^{-}e^{-i\Omega t} \qquad C_{12}^{\pm} = \frac{\mathcal{C}}{2}\int_{\mathrm{MA}} d\mathbf{r} E_1^* E_2[\delta_R e^{\pm i\phi_R} \pm \delta_I e^{\pm i\phi_I}]$$

❑ Coupling to external ports:



$$s_2^{\mathrm{out}}(t) = \xi_2 s_2^{\mathrm{in}}(t) + \kappa_2 a_2(t)$$

$$s_1^{\mathrm{out}}(t) = \xi_1 s_1^{\mathrm{in}}(t) + \kappa_1 a_1(t)$$

$$\boxed{\begin{aligned} \dot{a}_1(t) &= (i\omega_1 - \gamma_1)a_1(t) + i[C_{12}^{+}e^{i\Omega t} + C_{12}^{-}e^{-i\Omega t}]a_2(t) + \kappa_1 s_1^{\mathrm{in}}(t) \\ \dot{a}_2(t) &= (i\omega_2 - \gamma_2)a_2(t) + i[C_{21}^{+}e^{i\Omega t} + C_{21}^{-}e^{-i\Omega t}]a_1(t) + \kappa_2 s_2^{\mathrm{in}}(t) \end{aligned}}$$

$$a_{1,2}(t) = a_{1,2}^{H}(t) + a_{1,2}^{P}(t)$$

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Homogeneous solution: Floquet theory

❑ Floquet theory for time-periodic systems:

$$\dot{\mathbf{x}}(t) = \mathbf{O}(t)\mathbf{x}(t)$$
with $\mathbf{O}(t) = \mathbf{O}(t+T)$ $\Rightarrow$ $\mathbf{x}(t) = e^{-i\mu t}\mathbf{p}(t)$ where $\mathbf{p}(t) = \mathbf{p}(t+T)$, μ quasi $-$ energy

The quasi-energies are solutions to $\det[\mathbf{U}(T,0) - e^{-i\mu T}\mathbf{I}] = 0$ $\mathbf{x}(T) = U(T,0)\mathbf{x}(0)$

❑ For our modulated meta-atom

$$a_{1,2}^{H,\pm}(t) \sim e^{-i\mu_{\pm}t}\, e^{-\gamma_{1,2}t}$$

$$\boxed{\mu_{\pm} = \frac{-(\omega_1+\omega_2) \pm \sqrt{(\omega_2-\omega_1-\Omega)^2 + 4C_{12}^{+}C_{21}^{-}}}{2}}$$

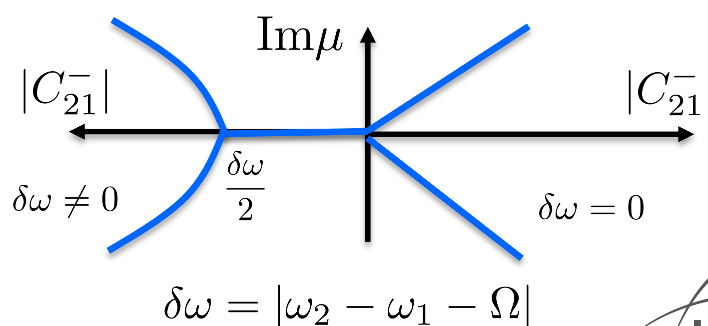
When $(\omega_2-\omega_1-\Omega)^2 + 4C_{21}^{+}C_{12}^{-} > 0$, both quasi-energies are real, and hence $a_{1,2}^{H,\pm}(t) \to 0$

Otherwise, $\mu_{\pm}$ has an imaginary component → exponential growth in time (gain)

❑ Example: modulate only $\mathrm{Im}(\delta\epsilon)$

$$\Rightarrow C_{12}^{+} = -(C_{21}^{-})^{*}$$

$$\mathrm{Im}\mu_{\pm} = \pm\frac{1}{2}\sqrt{(\delta\omega)^2 - 4|C_{21}^{-}|^2}$$



Nonreciprocal metasurfaces

- In steady state $s_1^{\text{in,out}}(t) = \mathcal{S}_1^{\text{in,out}} e^{i\omega t}$ $\quad s_2^{\text{in,out}}(t) = \mathcal{S}_2^{\text{in,out}} e^{i(\omega+\Omega)t}$

$$a_{1,2}^{H,\pm}(t) \to 0 \qquad a_1^P(t) = \mathcal{A}_1\ e^{i\omega t} \qquad a_2(t) = \mathcal{A}_2\ e^{i(\omega+\Omega)t}$$

$$\dot{a}_1(t) = (i\omega_1 - \gamma_1)a_1(t) + iC_{12}^{-}e^{-i\Omega t}a_2(t) - \kappa_1 s_1^{\text{in}}(t)$$
$$\dot{a}_2(t) = (i\omega_2 - \gamma_2)a_2(t) + iC_{21}^{+}e^{i\Omega t}a_1(t) - \kappa_2 s_2^{\text{in}}(t)$$

where we used the rotating-wave approximation (discard fast counter-rotating terms)

- Output fields

$$\boxed{\begin{aligned}\mathcal{S}_1^{\text{out}} &= \left\{\xi_1 + \frac{\kappa_1^2[i(\omega-\omega_2)+\gamma_2]}{D}\right\}\mathcal{S}_1^{\text{in}} + \frac{i\kappa_1\kappa_2}{D}C_{12}^{-}\mathcal{S}_2^{\text{in}} \\ \mathcal{S}_2^{\text{out}} &= \left\{\xi_2 + \frac{\kappa_2^2[i(\omega-\omega_1)+\gamma_1]}{D}\right\}\mathcal{S}_2^{\text{in}} + \frac{i\kappa_1\kappa_2}{D}C_{21}^{+}\mathcal{S}_1^{\text{in}}\end{aligned}}$$

$$D = [i(\omega-\omega_1)+\gamma_1][i(\omega+\Omega-\omega_2)+\gamma_2] + C_{12}^{-}C_{21}^{+}$$

When $C_{12}^{-} \neq C_{21}^{+} \ \Rightarrow$ System is nonreciprocal

- "Partial" nonreciprocity $\delta_R \neq 0\ ,\delta_I = 0$

$$C_{12}^{-} = (C_{21}^{+})^* \qquad \boxed{S_{12} \sim e^{-i\phi} \neq S_{21} \sim e^{+i\phi}}$$

- "Full" nonreciprocity $\delta_R = \delta_I$ and $\phi_R = \phi_I$

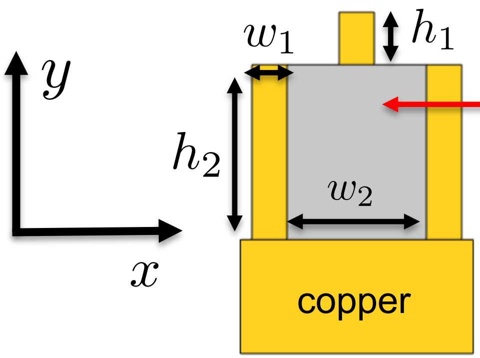
$$\boxed{S_{12} = 0 \neq \quad S_{21} \sim e^{+i\phi}} \qquad C_{21}^{+} \neq 0 \qquad C_{12}^{-} = 0$$

Dynamic (nonreciprocal) focusing with STMMs

- Phase distribution over the meta-surface $\phi = \phi(x)$

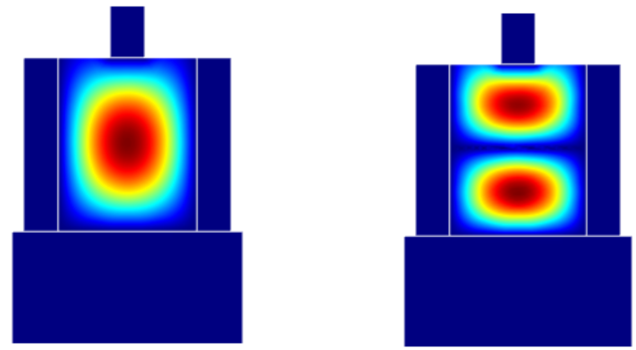
→ dynamical wavefront control using spatio-temporal modulated metasurfaces

- Unperturbed meta-atom



$$\epsilon_c = 12.25$$
$$h_1 = 3 \text{ mm}$$
$$h_2 = 10 \text{ mm}$$
$$w_1 = 2 \text{ mm}$$
$$w_2 = 8 \text{ mm}$$

$\omega_1 = 6.8$ GHz $\qquad$ $\omega_2 = 9.9$ GHz

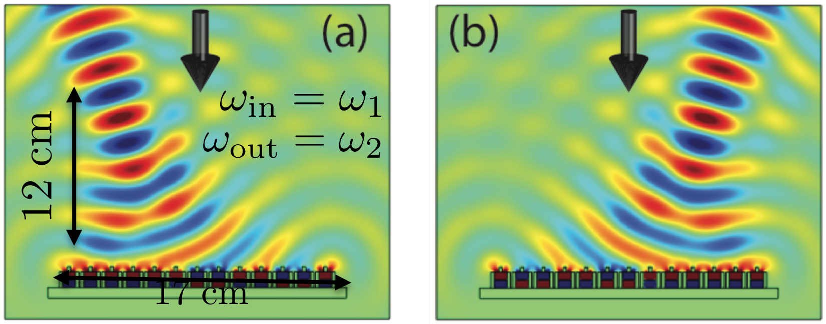


- STMM with parabolic phase distribution

$$\epsilon(x, y, t) = \epsilon_c + \delta(y)\cos[\Omega t + \phi(x)]$$

$$\phi(x) = \frac{\omega_{\text{in}}}{c}\sqrt{(x - x_f)^2 + y_f^2} \qquad \Omega = \omega_2 - \omega_1$$

$$\delta(y) = \delta\,\theta(y - h_2/2) \qquad \delta/\epsilon_c = 0.1$$



$x_f = -5$ cm $\qquad$ $x_f = +5$ cm

Dynamic (nonreciprocal) beam steering with STMMs

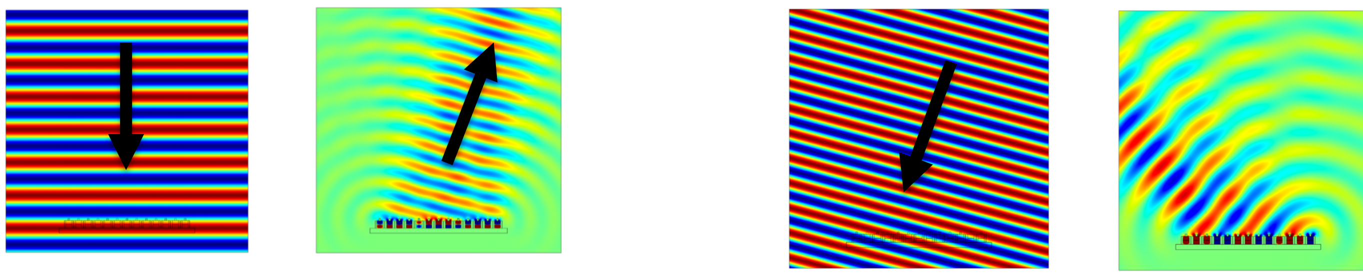
- **STMM with linear phase distribution** $\phi(x) = \frac{\omega_{\text{in}}}{c} x \left(\sin\theta_i - \sin\theta_r\right)$

- Modulating the real part only

$$\epsilon(x,y,t) = \epsilon_c + \delta(y)\cos[\Omega t + \phi(x)] \implies S_{12} \sim e^{-i\phi} \neq S_{21} \sim e^{+i\phi}$$

$\theta_i = 0$ deg

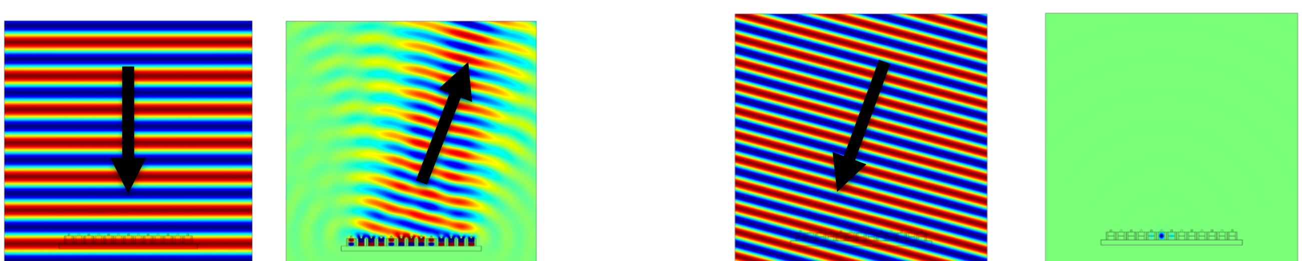
$\theta_r = 15$ deg



$\omega_{\text{in}} = \omega_1$ $\omega_{\text{out}} = \omega_2$ $\omega_{\text{in}} = \omega_2$ $\omega_{\text{out}} = \omega_1$

- Modulating both the real and imaginary parts

$$\epsilon(x,y,t) = \epsilon_c + \delta(y)\left\{\cos[\Omega t + \phi(x)] + i\sin[\Omega t + \phi(x)]\right\} \implies S_{12} = 0 \neq S_{21} \sim e^{+i\phi}$$



$\omega_{\text{in}} = \omega_1$ $\omega_{\text{out}} = \omega_2$ $\omega_{\text{in}} = \omega_2$ no output at ω_1

Summary and future work

- Electromagnetic nonreciprocity: a tricky subject!
- Coupled mode theory for nonreciprocal STMMs: asymmetric scattering matrix. Dynamical (nonreciprocal) focusing and beam steering
- Paper in preparation

Our next steps in the theory developments:

- Model and design suitable time-Floquet meta-atoms for experiments on nonreciprocity $\epsilon(x,y,t) = \epsilon_c + \delta(y)\cos[\Omega t + \phi(x)]$
- Consider travelling wave perturbations $n(x) = n_0 + \delta n \cos(K_{\mathrm{MS}} x + \omega_{\mathrm{MS}} t)$ using Bloch-Floquet theory. Model STMM for experimental demonstration of nonreciprocal wavefront control with this type of perturbations
- Study other kinds of “abrupt” perturbations, e.g. “coding” STMMs
- Explore PT symmetric metasurfaces for wavefront control using gain/loss